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BUCKLING TESTS OF LIGHT-METAL TUBES

By August Schroeder

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BUCKLING TESTS OF LIGHT-METAL TUBES.*

By August Schroeder.

I will attempt to determine mathematically the buckling-strength curves of various centrally loaded light-metal tubes which exhibit conspicuous differences of behavior under compressive loads. For this purpose, I will employ Von Karman's method, after adapting it to the special conditions.

There is generally considerable uncertainty in the calculation of structural members subjected to buckling stresses when these members, due to their small slenderness ratio, do not come within the range of validity of Euler's formula, which is known to cease when the stresses developed in the member exceed the limit of elasticity of the member. For the wider domain of unelastic buckling, empirical formulas (like those of Tetmajer) are almost exclusively used. In these formulas the curve of the buckling strengths in this field is estimated as closely as possible with the aid of experimentally determined coefficients. Thus a straight line or a simple curve is formed on Euler's hyperbola. The coefficients required for the calculation are usually taken from handbooks. These coefficients are valid, however, only for materials which behave the same under compressive stresses as the material for which they were determined.

*"Druck- und Knickversuche mit Leichtmetall-Rohren." From the 1928 Yearbook of the Deutsche Versuchsanstalt für Luftfahrt," pp. 216-222.

In all these formulas no account is taken of the cross-sectional shape of the members, which, in many cases, greatly affects the magnitude of the attainable buckling stresses.

The characteristic differences between the compression curves for different light metals, as found in compression tests with short tubes (Fig. 1), furnished the reason for an investigation as to the possibility of determining the buckling strength curves for these materials and for this form of girder. The method for this investigation is supplied by the researches of Von Karman on buckling strength, in which he sought to determine the buckling strength curves on the basis of the behavior of different materials when subjected to simple compression.* For centrally loaded members, Von Karman developed a formula for the calculation of the buckling load from the consideration of the stress distribution over the cross section of the member in the immediate vicinity of the straight condition. It is obvious that the cross-sectional shape must be considered, since this affects the results of the calculation to a certain degree. It is questionable, however, whether the thus-determined dependence of the buckling strength on the cross-sectional shape can be regarded as satisfactory, when the cross sections are not stiff enough for the member to buckle as a whole (i.e., without materially affecting its cross-sectional shape). Open or half-open thin-walled profiles, as used in airplane construction, do not meet these conditions. To a greater or less degree the

*Von Karman, "Untersuchungen über Knickfestigkeite," Forschungsarbeiten des V.D.I., No. 81.

buckling of the member is always brought about by the buckling of some particular part of the cross section. This kind of consideration of the cross section is not sufficient even for the normal tubes used in airplane building since, for large spans, changes in shape occur, which are not due alone to the elastic or plastic deformability of the material, but in part to the deformability of the test specimen as a whole.

Therefore it seems necessary to try the compression tests required for the calculation of the buckling strength curves on the basis of Von Karman's theory, not on prismatic or cylindrical specimens, but on specimens which have the same cross-sectional shapes and dimensions as the members under investigation.

There is still another reason for making such compression tests. Due to the production of profiles or tubes by drawing, pressing, rolling, stamping, etc., the properties of the materials are often considerably changed. Hence it would probably be difficult to produce compressed objects of normal shape, which would correspond in their deformability to the material of the members under investigation. One particular difficulty consists in the fact that, due to the differing degrees of stress developed in the different cross sections during the production, the material can no longer be regarded as homogeneous.

Von Karman's theory will be discussed here only in so far as necessary for the analysis of the compression tests and the further calculation. Moreover, reference must be made to the

treatise itself, whose fundamental ideas are somewhat as follows. If we consider a member at first uniformly compressed and then slightly bent, we obtain a stress distribution like Figure 2 on the assumption that:

1. With a slight bend of a straight member, the elongations (or shortenings) of the individual fibers correspond to the same stresses (even beyond the limit of elasticity) which produce these elongations in a simple tensile or compressive test.

2. The elongations (or shortenings) of the fibers of a slightly bent member can be approximately calculated on the assumption of the Karman theory that "plane cross sections remain plane."

That is to say, there is a "neutral axis" which is subjected to only the mean compressive stress. While, as a result of the bending, the compressive stress is increased on one side, it is diminished on the other. Correspondingly the law of deformation, as determined by the compression tests, holds good, on one side, for the total deformations of the material, while, on the other side, the law of the elastic deformations (which law is likewise determined by compression tests) must be used, since only the elastic deformations disappear on the removal of the load.

It is then demonstrated that, for centrally loaded members (under consideration of these circumstances), an extension of the application of Euler's formula is possible in the unelastic

field, by introducing a special modulus of buckling M instead of the modulus of elasticity E. This modulus M forms a mean value between the two moduli M_1 (total deformations) and M_2 (elastic deformations) to be determined by compression tests. The manner of finding the mean value from M_1 and M_2 depends on the shape of the cross section. For this purpose the following instructions generally apply.

The cross section must be divided into two parts by a straight line parallel to the axis of the minimum moment of inertia, so that between the static moments of these parts and the moduli M_1 and M_2 we have the relation $M_1 S_1 = M_2 S_2$, which means that the resultants of the mutual stress increments offset one another. If the inertia moments of the two parts of the cross section with respect to this straight line are designated by J_1 and J_2, and the inertia moment of the whole cross section by J, we then have

$$M_s = J_1 M_1 + J_2 M_2$$

for the moment of the stresses, or if we put $M_s = J M$, we obtain

$$M = \frac{J_1}{J_1 + J_2} M_1 + \frac{J_2}{J_1 + J_2} M_2 .$$

Tubes of four different light-metal alloys were investigated. Tensile tests were first made with specimen tubes. The yield point $(\sigma_{0.2})$* and the modulus of elasticity (E) were

*That is, the point at which the permanent elongation is 0.2%. See "Hütte," Vol. I, p.579.

determined with the aid of Baumann's extensometer, which has an accuracy of 1/5000 cm (0.002 in.). The alloys differed in their chemical composition, manner of production and thermal treatment. The results of the tensile tests are given in Table I.

TABLE I. Tensile Tests

Alloy No.	Yield point $\sigma_{0.2}$ kg/mm²	Breaking stress σ_B kg/mm²	Elongation at rupture δ_{10} %	Modulus of elasticity E kg/cm²
1	27.0	40.0	18.0	730,000
2	22.1	31.0	3.2	430,000
3	19.5	30.5	10.0	425,000
4	15.5	29.0	15.4	425,000

As regards these results, it is noticeable that the modulus of elasticity of alloys 2 to 4 has the given value only up to stresses which are in part far below the yield point $\sigma_{0.2}$ and then diminishes with comparative rapidity, while it remains constant for alloy 1 nearly up to the breaking stress. In Figure 3 the tensile force is plotted against the elongation (total deformation) up to the yield point.

The dimensions of the tubes from which the specimens were taken for the compression and buckling tests are given in the approximate tables. The straightness and roundness of the tubes was satisfactory. In some of them there were considerable variations in the thickness, which may greatly affect the homogeneity of the material. For example, the values in Table II were obtained with tubes of different thicknesses (alloy No. 4), having the same external diameter. These values indicate the unequal strength of the material.

TABLE II. Tensile Tests with Members of Different Thickness

Specimen No.	Thickness s mm	Yield point $\sigma_{0.2}$ kg/mm²	Breaking stress σ_B kg/mm²	$\frac{\sigma_{0.2}}{\sigma_B} \times 100$ %	Elongation at rupture δ_{10} %
1	1.91	14.7	28.8	51	14.8
2	1.70	16.2	29.4	55	13.8

The relations are similar in compressive stresses, whereby the centering of the test specimen is rendered more difficult, and the maintenance of the centering, after a certain critical stress is exceeded, becomes impossible under certain conditions.

Compression tests with short tubes.- The following points must be given special consideration. Differences in the material, irregularities in thickness, etc., render impossible the centering of the test specimen simply according to the outside dimensions. While it is possible with relatively slender tubes to effect a sufficiently accurate centering during the test, by observing the direction of buckling, it is attended by difficulties in the case of large tubes and especially of short tubes. The results of compression tests, made between plates resting on balls, may therefore be unfavorably affected by the original eccentricity to an extent not to be overlooked. If, on the other hand, the compression tests are made between fixed plates, we obtain too favorable results in the field of the higher stresses, since, even with the most accurate original centering, it is impossible in practice to maintain the central position of the load beyond a certain stress (compressive yield point), while this is effected automatically

to a certain extent between plates resting on balls. In order to investigate these relations more thoroughly in the case under consideration, compression tests were made with short tubes both between fixed plates and between plates resting on balls.

The test specimens were tubes having a length to diameter ratio of 1.5 and 2, it being understood that the effect of the length of the specimens could not be determined by these tests. In preparing the test specimens, special care had to be taken to make the end surfaces of each tube exactly parallel to each other and perpendicular to the axis of the tube. The surfaces on both ends were wiped clean.

The tests were made between the plates of a twenty-ton testing machine made by Mohr and Federhaff of Mannheim. For the tests between plates resting on balls, special compression pieces resting on 10 mm (0.394 in.) balls were inserted. The record of the compression curves was made by measuring the compression over a length of 5 cm (1.97 in.) with the aid of a Baumann extensometer and, furthermore, the compression of the whole cross section by means of a Zeiss dial to 0.01 mm (0.0004 in.). The two measurements differed in that the total deformation of the whole tube was measured with the latter instrument, while the properties of the material itself were determined by measuring over a relatively small length with the Baumann extensometer. The mounting of the test specimens in the testing machine is shown in Figures 4 and 5.

The method of testing and the results were as follows. The measurements made with the extensometers generally showed no essential differences, up to relatively high stresses, in comparative tests with fixed plates and with plates resting on balls. The curves in Figure 6 plainly show the deviations which differed in magnitude according to the accuracy of the original centering in the other tests.

The difference between the measurements over the whole length of the test specimens and over a portion of the length first appeared at stresses which produced permanent deformations. The curves in Figure 7 show the characteristic difference for these methods of testing. These show that the compression test between plates resting on balls may be unfavorably affected by original inaccurate centering and that the measurement over a given part of the length yields too favorable results, since they are only slightly affected by the deformability of the test specimens. The results obtained with the Zeiss dials in the compression tests between fixed plates were accordingly used for the evaluation, whereby it was to be expected that the calculations in the region of the high stresses would yield too favorable results.

In connection with the evaluation, it should also be noted that the pointers of the dials fluctuate greatly in the elastic region, since the decrements to be measured are very small in proportion to the measuring accuracy. Since, however, in the

elastic region the deformations of the test specimen itself have no appreciable effect on the measured decrements, the values obtained with the extensometers could be introduced here. The two series of tests can be easily combined, since we are not concerned with the absolute values of the decrements, but only with their increase for one stress stage.

Figure 8 shows the different forms of rupture, while Figure 1 shows the characteristic deformation curves for each material. The reproduction of the individual test curves was omitted.

Mathematical determination of the deformation moduli M_1 and M_2.- The modulus of deformation (like the modulus of elasticity) for a given material furnishes a criterion for the deformability of a body at a given stress and denotes the same as the trigonometrical tangent at the point of the compression curve corresponding to this stress. The determination of this modulus for the elastic deformation (M_2) and for the total deformation (M_1), is therefore very simple. For the different stages of stress (at which the load was removed two or three times at intervals), this calculation was made for the curves of the elastic and of the total deformation according to the results of the compression tests, and the calculated modulus was coordinated with the mean stress of this stage.

In Figure 9 the variations of M_1 and M_2 are plotted against the stress for the four alloys. The curves show that

not only M_1 but also M_2 decrease quite rapidly even at relatively small stresses. The decrease in the modulus of the elastic deformations M_2 for the alloys 2, 3, and 4 represents, in part, a property of the material and, in part, a property of the test specimen while, for alloy 1, the decrease is probably due to the deformability of the test specimen. This is quite well demonstrated by the fact that, for this alloy, in testing a short section with Baumann's extensometer, even in the compression test, the modulus M_2 remains fairly constant (in agreement with the tensile test) nearly to the breaking point. The separate curves were plotted with the closest possible approximation to the test points (which were always somewhat scattered). It is still uncertain as to how far the curves correspond in detail to the actual behavior of the test. In comparison with the fundamental course of the curves, the deviations which, under some conditions, are due to test errors, are nevertheless small and affect but slightly the results of the further calculation.

<u>The calculation of the resultant modulus M for tube cross sections and the mathematical determination of the buckling-strength curves</u>.- For the determination of M it is expedient to draw first for the investigated cross section (of the tube in this case) several curves which represent graphically the relation of the static moments S_1 and S_2 in the displacement of

the reference axis and likewise the values of the partial inertia moments with reference to the axis. Since the analysis requires no absolute values, this representation cannot be considered quite general, i.e., independent of the diameter of the tube and the thickness of its walls. The partial inertia moments are therefore put in relation to the total inertia moments over the axis of gravity. Figures 10 and 11 show these curves. I will illustrate the further course of the calculation by an example.

For alloy 1, according to Figure 9, the ratio

$$\frac{M_2}{M_1} = \frac{720000}{400000} = 1.8$$

corresponds to a tension of $\sigma_{-B} = 22$ kg/mm² (31300 lb./sq.in.). This number is to be put equal to S_1/S_2 and yields, according to Figure 10, a displacement of the "neutral" axis amounting to 17.5% of the radius. For this displacement, Figure 11 gives $J_1 = 0.75$ and $J_2 = 0.31$ for the two partial inertia moments and, since

$$M = \frac{J_1}{J_1 + J_2} M_1 + \frac{J_2}{J_1 + J_2} M_2$$

$$= \frac{0.75}{1.06} \times 400000 + \frac{0.31}{1.06} \times 720000$$

$$= 283000 + 211000 = 494000 \text{ kg/cm}^2$$

the expanded Euler formula gives

$$P_k = \pi^2 \frac{M\,J}{l^2} \quad \text{or} \quad \sigma_k = \pi^2 \frac{M}{(l/i)^2}$$

with the value

$$(l/i)^2 = \pi^2 \frac{M}{\sigma_k} = \frac{9.87 \times 494000}{2200} = 2215$$

$$l/i = 47.1$$

found for M. We can thus calculate for all stresses the value of l/i with which this stress can be attained and therewith the buckling-strength curves. Figure 12 shows the thus-calculated buckling-strength curves for the four different alloys. Figure 9 also shows the values for the resultant modulus M as intermediate between the modulus of the elastic deformations and the modulus of the total deformations.

With alloy 4 the following fact should be noted with respect to the curve M. Corresponding to the course of the compression curve (Fig. 1), a second increase in the modulus of buckling occurs after exceeding the clearly defined compressive yield point at which M is nearly equivalent to zero. This can likewise be followed mathematically. The buckling-strength curve does not then pass into the horizontal position, but proceeds somewhat like the dash line in Figure 12. The attainment of much higher buckling stresses was nevertheless not to be expected for a material with such a pronounced compressive yield point, since with the great deformation at this stress, even slight eccentricities (such as are always present) will cause a strong bending followed by rupture.

Similar critical points, though less pronounced, are shown

by the curves for the other alloys. For alloy 1, e.g., after the decrease of M between 22 and 23 kg/mm² (31300 and 32700 lb./sq.in.) becomes quite large, this decrease again diminishes and the curve bends strongly at about 23 kg/mm² (32700 lb./sq.in.). This is mainly attributable to the supporting effect of the fixed plates. In an actual buckling test, we must therefore expect, from this point on, a deviation of the experimental buckling curve from the calculated one. In general, the deviations will be larger, the more pronounced the bend in the M curve and the steeper the descent of M just before the turning point.

For alloy 2, which has only a slight deformability (elongation at rupture $\delta_{10} = 3.2\%$ in the tensile test), the critical point is very high, 22 to 23 kg/mm², while it is 16 to 17 kg/mm² (22750 to 24180 lb./sq.in.) for alloy 3. For alloy 4, as already mentioned, this point is represented by the limit of elasticity. The change in direction and the previous drop in the curve are here particularly great.

Buckling tests with alloys 1 and 4.- These tests were made on the same machine as the compression tests. Even the fixation clamps were the same as in the compression tests between plates resting on balls. The tubes were held against the compression plates by means of clamps which could be adjusted by screws in two directions perpendicular to one another. It was thus possible under a small load, to center the tubes during the experiment by changing their position after observing the direction

of buckling during the first stages of loading. The direction and amount of buckling, to within 0.01 mm (0.0004 in.), was observed in two directions perpendicular to one another with the aid of a Zeiss dial. Figure 13 shows the manner of mounting the test specimens and the arrangement of the dials.

The centering was accomplished by alternate loading and unloading until, at stresses as near as possible to the limit of elastic deformability, the observed deflections were nearly or quite zero. Then the load was increased and the buckling load determined. In all cases the buckling load was defined as the load at which the deflection continuously increased without increasing the load. A certain moment of fixation was naturally produced in the ball bearings. The effect of this relatively slight fixation on the buckling load is extremely small at a small l/i ratio, so that deflections from this cause are to be expected only in the Euler region.

The lengths of the test specimens were so graduated that the whole range of buckling-strength curves was investigated from slenderness ratios of $l/i = 120$ down to $l/i = 15$. In cases where the compressive stresses remain small and the straightness of the tube is not affected, the same tube was shortened for further experiments with a smaller l/i ratio. Parallel tests were made with one test specimen of each alloy. Since the results agreed very well and since deviations from the computed curves occurred only in the anticipated direction,

further parallel tests were omitted. The dimensions of the test specimens are given in Tables III and IV along with the numerical results of the tests.

In connection with the results obtained with short tubes of alloy 4, it should be mentioned that still higher stresses were reached at a slenderness ratio of less than 20, after very large deflections had occurred at the critical stress. The resistivity of any one of these tubes was fully exhausted only with the occurrence of a local buckling in the wall of the tube. These values are of no importance, however, in practice, since inadmissibly large deformations occur previously. The stresses thus found quite closely approximate the dash curve.

TABLE III. Compression Tests with Alloy 1. Dimensions of Test Specimens. Test Results.

No.	Dimensions mm	Cross section F mm²	Buckling length l cm	l/i	Breaking load P kg	Buckling stress σ_k kg/mm²
1	50 x 1.6	244	207	121	1390	5.7
2	50 x 1.6	244	147	86	2800	11.5
3	80 x 2.6	490	207	75	6800	13.9
4	50 x 1.6	244	107	62.5	4290	17.6
5	50 x 1.6	244	77	45	5540	22.7
6	50 x 1.6	244	57	33.3	5770	23.6
7	50 x 1.6	244	57	33.3	5700	23.4
8	50 x 1.6	244	32	18.7	6370	26.1

TABLE IV. Compression Tests with Alloy 4.
Dimensions of Test Specimens.
Test Results.

No.	Dimensions mm	Cross section F mm²	Buckling length l cm	l/i	Breaking load P kg	Buckling stress σ_k kg/mm²
1	60 x 2.08	378	257	125	1300	3.44
2	80 x 1.77	435	259	93.5	2390	5.50
3	80 x 1.77	435	197	71	3100	7.13
4	50 x 1.77	268	107	63	2120	7.91
5	50 x 2.26	340	108	64	2730	8.03
6	50 x 2.26	340	77	45.5	3090	9.10
7	50 x 1.76	266	43	25.0	2470	9.29
8	50 x 1.76	266	25	14.5	2460	9.25(11.7)
9	50 x 1.77	268	21.5	12.5	2450	9.15(13.3)
10	50 x 1.77	268	19	11	2440	9.15(14.9)

Figure 12 shows the test points for both alloys together with the computed buckling-strength curves. The agreement between the test results and the computed curves is entirely satisfactory, since the discrepancies can be explained. In the region of the small l/i ratios the curve can, with a little practice, be easily verified by a very small number of tests.

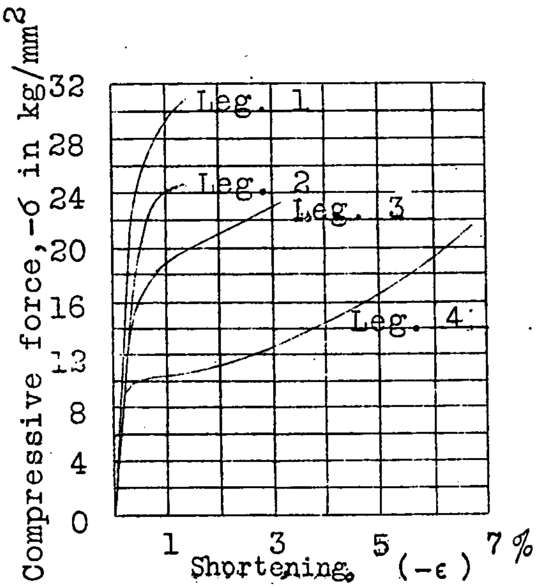
Summary

From the tests we find it possible to calculate with sufficient accuracy the buckling-strength curves for tubes from the most widely differing light-metal alloys (with central loading), on the basis of their reaction to simple compression (corresponding to the researches of Von Karman) if, in addition to the deformability of the material, we also take into account the deformability of the test specimen itself.

This was accomplished by making, with test specimens, the compression tests required for the calculation of the buckling-strength curves, the test specimens corresponding in their cross-sectional shapes and dimensions to the compression members under investigation. For tubes (and probably for all members having closed cross sections of symmetrical form) it appears permissible, for the sake of simplicity, to make the compression tests between fixed plates, whereby, however, it must be remembered, in every case, that the results of the calculation will be too favorable above a certain stress (more or less definite compressive yield point), which is characteristic of the given material.

Comparison of the compressive and tensile tests shows that the choice of the material for compression members on the basis of the results of tensile tests may, under certain conditions (alloy 4), lead to very wrong conclusions.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

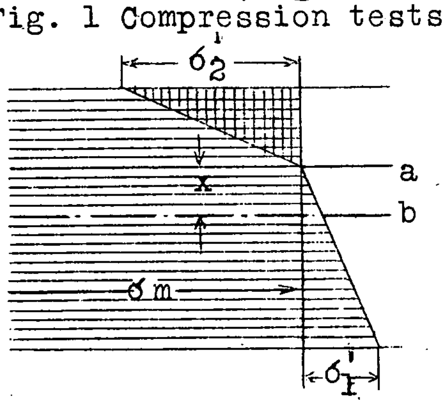


Buckling-strength curves of light-metal tubes (length of specimen h=1.5 d). Total deformations. Apparatus: 2 Zeiss dials. Accuracy: 0.01 min.(.0004 in.)

Breaking stresses:

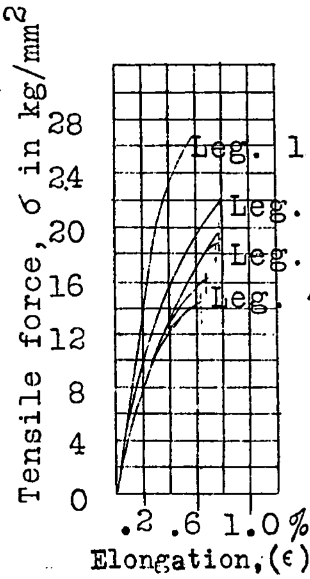
Leg.(Alloy) 1, σ_{-B}=32.5 kg/mm^2
" " 2, σ_{-B}=27.0 kg/mm^2
" " 3, σ_{-B}=24.2 kg/mm^2
" " 4, σ_{-B}=27.6 kg/mm^2

Fig. 1 Compression tests.



a-Neutral axis
b-Axis of member

Fig. 2 Stress distribution in a member uniformly compressed and then slightly bent.



Tensile-strength curves of proportional specimens of light-metal tubes.
Total deformations up to equal permanent elongations (yield point $\sigma_{0.2}$).
Apparatus:
Baumann extensometer.
Accuracy: 0.002 mm (.00008 in.).
Breaking stresses:

Leg.(Alloy) 1, σ_B=40.0 kg/mm^2
" " 2 σ_B=34.0 kg/mm^2
" " 3 σ_B=30.5 kg/mm^2
" " 4 σ_B=29.0 kg/mm^2

Parallel tests, with tubes of different thicknesses, of alloys 3 and 4 show the effect of different methods of production

Fig. 3 Tensile tests.

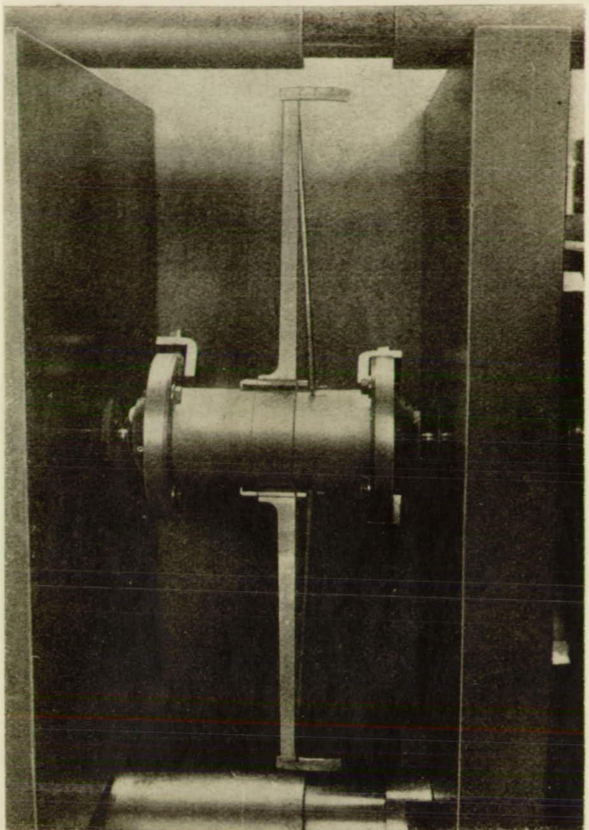


Fig.4 Compression test between rigid plates.

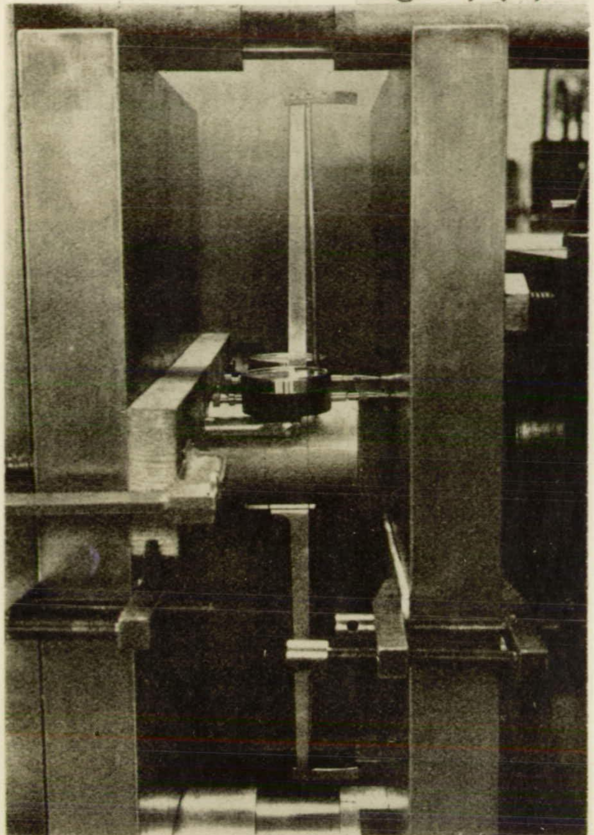


Fig.5 Compression test between plates resting on balls.

Fig.8 Forms of rupture.

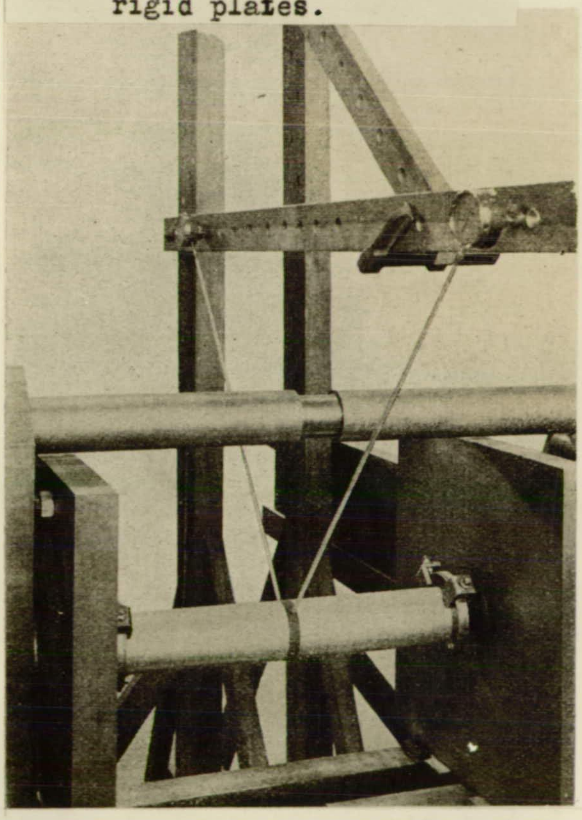


Fig.13 Testing machine with test bars installed.

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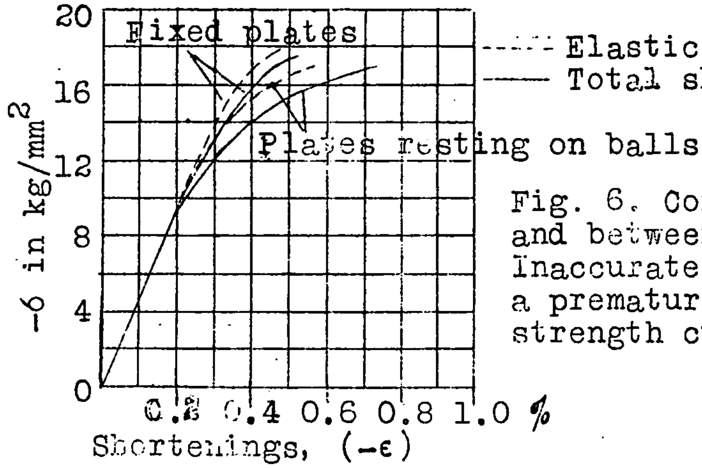


Fig. 6. Compression tests rigid plates and between plates resting on balls. Inaccurate original centering caused a premature bending of the buckling-strength curves.

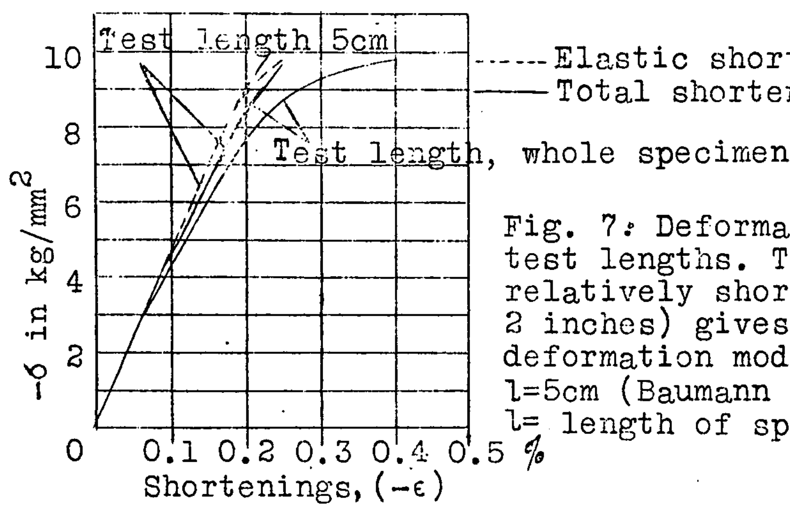
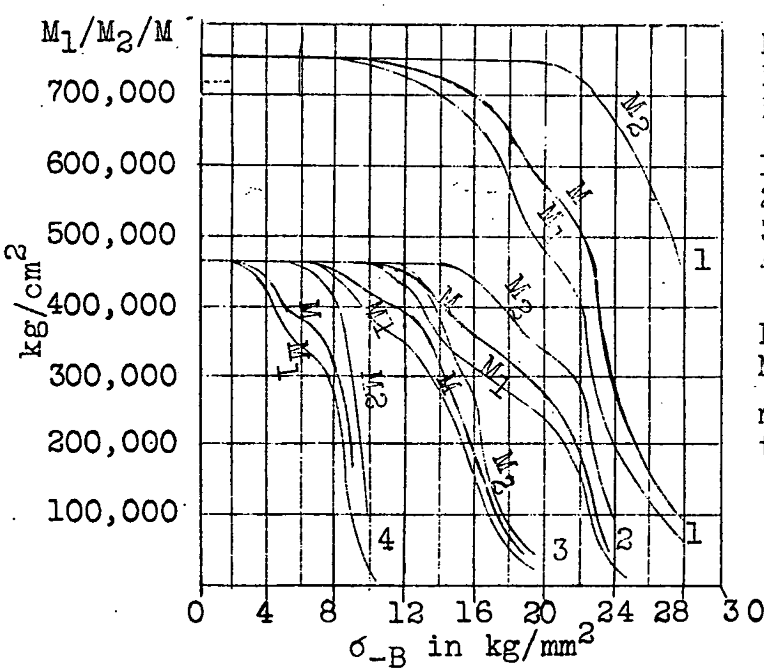


Fig. 7. Deformations for different test lengths. The test over a relatively short length (5 cm =about 2 inches) gives too favorable deformation moduli.
l=5cm (Baumann extensometer),
l= length of specimen (Zeiss dial).



M_2=Elastic M_2
M
M_1=Total M_1

1, Leg (alloy) 1
2, Leg (alloy) 2
3, Leg (alloy) 3
4, Leg (alloy) 4

Fig. 9. Deformation moduli M_1 and M_2, and buckling modulus M plotted against the compressive stress σ_{-B}.

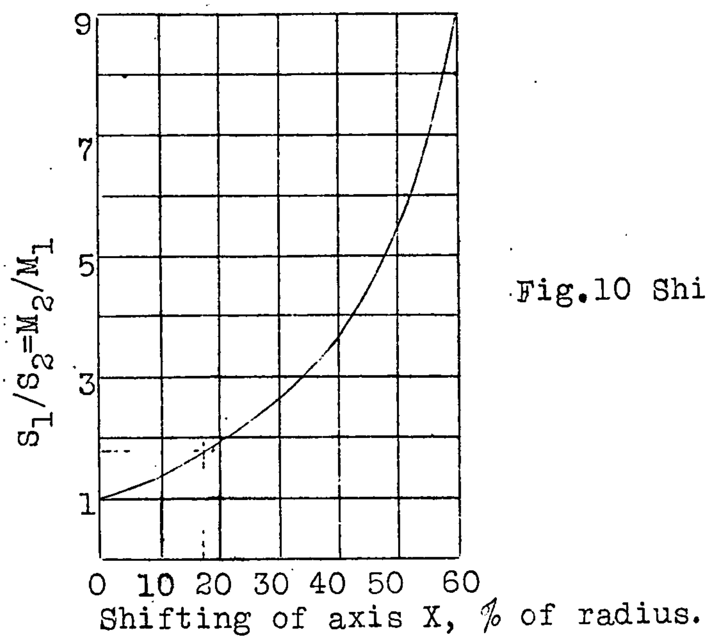


Fig.10 Shifting of the neutral axis.

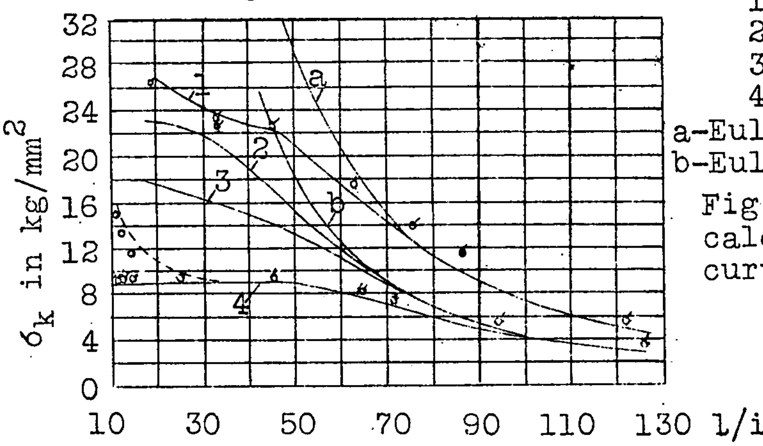


Fig.12 Test points and calculated buckling-strength curves.

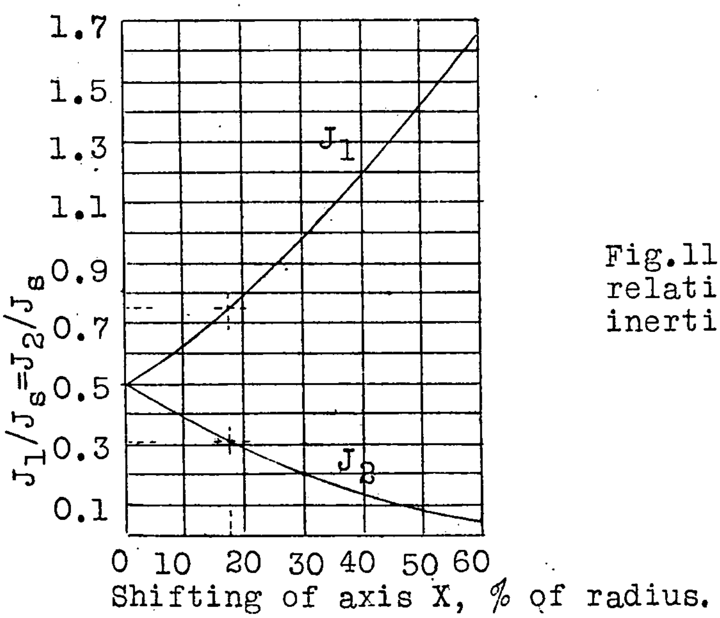


Fig.11 Determination of the relative values of the partial inertia moments J_1 and J_2.